Math 43 Quiz 1 Version 1
Thu Oct 5, 2017 (9:30am class)

NAME YOU ASKED TO BE CALLED IN CLASS:

SCORE: ____ / 30 POINTS

Write and **prove** a formula for $\sinh(x+y)$ in terms of $\sinh x$, $\sinh y$, $\cosh x$ and $\cosh y$. **SCORE: 6 / 6 PTS**

$\sinh(x+y) = \boxed{\sinh x \cosh y + \cosh x \sinh y}$ 1

~~sinl~~ ~~sinl~~ $\sinh x \cosh y + \cosh x \sinh y = \cancel{\left(e^{x} - e^{-x}\right)}$

$= \boxed{\left(\frac{e^{x}-e^{-x}}{2}\right)\left(\frac{e^{y}+e^{-y}}{2}\right) + \left(\frac{e^{x}+e^{-x}}{2}\right)\left(\frac{e^{y}-e^{-y}}{2}\right)}$ 1

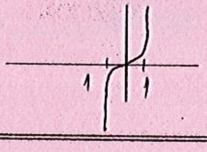
$= \frac{e^{x+y} + \cancel{e^{x-y}} - \cancel{e^{-x+y}} - e^{-(x+y)}}{4} + \frac{e^{x+y} - \cancel{e^{x-y}} + \cancel{e^{-x+y}} - e^{-(x+y)}}{4}$ 1 1 $\frac{1}{2}$

$= \boxed{\frac{2e^{(x+y)} - 2e^{-(x+y)}}{4}} = \boxed{\frac{e^{(x+y)} - e^{-(x+y)}}{2}} = \sinh(x+y)$ 1 $\frac{1}{2}$

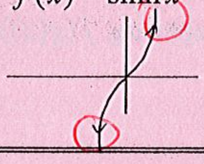
Sketch the general shape and position of the following graphs. **SCORE: $2\frac{1}{2}$ / 3 PTS**
(Don't worry about specific $x-$ or $y-$ coordinates.)

NO ASYMPTOTES

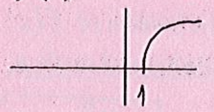
$f(x) = \tanh^{-1} x$



$f(x) = \sinh x$



$f(x) = \cosh^{-1} x$



Rewrite $\operatorname{sech}(\frac{1}{2}\ln 3)$ in terms of exponential functions and simplify. **SCORE: ____ / 3 PTS**

$\operatorname{sech}\left(\frac{1}{2}\ln 3\right) = \frac{1}{\cosh\left(\frac{1}{2}\ln 3\right)} = \boxed{\frac{2}{e^{\ln 3^{\frac{1}{2}}} + e^{-\ln 3^{\frac{1}{2}}}}} = \boxed{\frac{2}{\sqrt{3} + \frac{1}{\sqrt{3}}}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3+1} = \boxed{\frac{2\sqrt{3}}{4}} = \boxed{\frac{1\sqrt{3}}{2}}$

(*) 1 $\frac{1}{2}$ $\frac{1}{2}$

There is an identity involving $\sinh x$ and $\cosh x$ that resembles a Pythagorean identity from trigonometry. **SCORE: ______ / 7 PTS**

[a] Write that identity involving $\sinh x$ and $\cosh x$. **You do NOT need to prove the identity.**

$\boxed{\cosh^2 x - \sinh^2 x = 1}$ 1

[b] Write the identity for $\cosh 2x$ that uses both $\sinh x$ and $\cosh x$ simultaneously. **You do NOT need to prove the identity.**

$\boxed{\cosh 2x = \cosh^2 x + \sinh^2 x}$ 1

[c] Use the results of [a] and [b] to find and **prove** an identity for $\cosh 2x$ that uses only $\sinh x$.

$$\left.\begin{aligned}&\cosh^2 x - \sinh^2 x = 1\\ \to\ &\cosh 2x = \cosh^2 x + \sinh^2 x\end{aligned}\right\} \to$$

$\cosh 2x = \boxed{1 + \sinh^2 x + \sinh^2 x}$ 1 $\Rightarrow \boxed{\cosh 2x = \boxed{2\sinh^2 x + 1}}$ $\frac{1}{2}$

Prove: $\cosh 2x = 2\sinh^2 x + 1$

$= 2\left(\frac{e^{x} - e^{-x}}{2}\right)^2 + 1$

$= 2\left(\frac{e^{2x} - 2e^{0} + e^{-2x}}{4}\right) + 1$

$= \frac{e^{2x} + e^{-2x}}{2} = \cosh 2x.$

[d] If $\tanh x = -\frac{2}{3}$, find $\sinh x$ using identities.

You must explicitly show the use of the identities but you do NOT need to prove the identities.

Do NOT use inverse hyperbolic functions nor their logarithmic formulae in your solution.

$\tanh x = \frac{-2}{3} \Rightarrow \coth x = \frac{-3}{2}$ ✱

$\cosh^2 x - \sinh^2 x = 1$

$\to \frac{\cosh^2 x - \sinh^2 x}{\sinh^2 x} = \frac{1}{\sinh^2 x} = \operatorname{csch}^2 x$

$\to \coth^2 x - 1 = \operatorname{csch}^2 x \Rightarrow \left(-\frac{3}{2}\right)^2 - 1 = \frac{1}{\operatorname{csch}^2 x}$

$\Rightarrow \frac{1}{\operatorname{csch}^2 x} = \frac{5}{4} \Rightarrow \frac{1}{\operatorname{csch} x} = \frac{\sqrt5}{2} = \sinh x$

$\Rightarrow \sinh x = \frac{2}{\sqrt5}$

$\Rightarrow \frac{9}{4} - 1 = \frac{5}{4} = \operatorname{csch}^2 x$

$\to \operatorname{csch} x = \frac{\sqrt5}{2} \to \boxed{\sinh x = \frac{2}{\sqrt5}}$

Prove that $g(x) = \ln(x + \sqrt{x^2 - 1})$ is the inverse of $f(x) = \cosh x$ by simplifying $f(g(x))$. **SCORE: 1 / 5 PTS**

You may need to use the exponential definition of $\cosh x$.

$f(g(x)) = \ln(\cosh x + \sqrt{\cosh^2 x - 1})$

$= \ln(\cosh x + \sqrt{\sinh^2 x})$

$= \ln(\cosh x + \sinh x)$

$= \ln\left(\frac{e^{x} + e^{-x} + e^{x} - e^{-x}}{2}\right)$ 1

$= \ln\left(\frac{2e^{x}}{2}\right) = \ln e^{x} = \boxed{x.} \Rightarrow g(x)$ is the inverse of $f(x)$.